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Estimation of age dependent expected accomplishment probabilities in a population

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Abstract
This paper proposes and present expected accomplishment probability model during each of the three age epochs in a population. Methods for the estimation of these probabilities are also proposed and presented. Test statistic are proposed and presented for use in testing desired hypotheses about expected accomplishment probabilities. The proposed methods are illustrated with some sample data on retirees from a certain population in which human life or age span is partitioned into three, namely, less than 25 years, 25 to 50 years, and over 50 years. The results obtained using these sample data showed that less than one-fourth of retirees from the sampled population are able to accomplish all that is normally expected of subjects from the population after retirement from relatively active work. However, only about one-fifth of retirees seem unable to accomplish all that is normally expected following retirement.

Keywords: Accomplishment Probability, Age dependent, Conditional Probability, Expected Probability, Hypothesis

Introduction

It has been stated and argued that one of the ways to ensure the promotion of sustained individual and national social and economic growth and development in both developed and developing populations is to inform, educate and make the populace fully aware that in the process of growing up in these populations, it is necessary for one to know and be conscious that as teen you can afford to relax; but at 25, you should have made some waves, caused some stir and stare; at 30, you should be at your peak; at 40, you are probably a late starter; at 50, you may need to change direction; avoid burn-out; at 60 you should be ready to retire; at 70 you should be prepared to start another life; relax; at 80 you thank all concerned and go back to the village; the young shall grow (Jones, 2010)

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This growth model, it is hoped would perhaps be useful to psychologists, sociologists, planners, politicians, leaders, decision makers in a population for planning and remediation for interventionist purposes.

This growth model, it is hoped that it would also help modulate and redirect the often time-filtrating individual, group and national life styles, sometimes on a random drift, especially for the younger generation, in some populations who fortunately may still have all the options to organize and plan own life. Planning would enable one reduce the chances of mistaken decisions and their deleterious effects on quality of life. Without adequate and proper planning one often covers the earth with own mistakes until the earth, irritated, prematurely covers one up (Jones, 2009).

It is important to note that, whether one is in public service, self-employed or otherwise engaged in some income yielding enterprise, given the fullness of life, one would ultimately retire and take a break from active economic activities. Retirement and sustained break from active economic productivity is a phase of lifecycle one should normally look forward to, given proper planning, organization, and management at both personal and societal levels (Ng and Feldman 2013). Thus, ordinarily one should be able to eagerly anticipate retirement and look forward to finally having the time and freedom to do whatever one chooses and spend more time with family and friends pursue cherished hobbies, travel, community and volunteer services, or simply take life a little bit easier.

However, achieving these personal fulfillments in retirements is considered possible only if one had ab-initio made adequate plans during one's more active life. Unfortunately, there is increasing tendency for working people for example pro-offer excuses against savings and investments (Farrell and Finkelstein, 2007).

For instance, people offer many reasons for not saving additional funds for the future and for retirement including such excuses as "I will start saving next year", "My kids will take care of me", "I am too old to start saving", "I am too young to think about retirement", "My pension will give me enough to live on", etc.

However, no matter one's personal circumstances, it can always seem difficult to find the funds or the motivation and time to plan for the future both financially or otherwise (Weinberg and Galenson, 2005).

We in this paper present how to statistically model some expected accomplishments over a life time and the estimates of associated probabilities. In other word, we here purpose and develop a statistical model for use in the estimation of the probabilities that a randomly selected subjects from a population achieves or

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accomplishes some social and economic status that subjects from such a population would normally be expected to attain at various ages in the population. The method would enable one estimate the chances of accomplishing or attaining some status normally expected of one at various stages in life and perhaps better illuminate the social economic factors where such age dependent self-actualization is more likely to be achieved. The model would also perhaps provide some insight and information for psychologists, sociologists, planners, policy and decision, makers in a population to use for counseling and guidance in formulating targeted informed necessary and appropriate for the interventionist measures for the population. Test statistics are developed and presented for testing any desired hypotheses about expected accomplishment probabilities.

METHODOLOGY

A person's life is interspersed by milestones of achievements, failures, successes, disappointments and accomplishments right from ones birth through ones working life down to one's death. This continuum of life would however in this paper for simplicity, be partitioned into three rather broad groups namely the dependent and relatively the inactive population or group due to being under age or too young to work; the economically active population or group; and the dependent and relatively inactive population or group as a result of old age or increasing age (Struthers et al, 1996)

These three groups actually differ in their actual age classifications depending on the population being studied. In most developed and industrialized populations, the two dependent population groups may be respectively those who are less than 18 years and those over 70 years. However, in some developing populations, the under age and young dependent age group may include those as old as 24 years or more who may still be in school or not working because of high unemployment. The economically active population group may here be people in their mid-twenties up to age 60 or 65 years; the rest may be retirees and relatively inactive group due to advancing age.

Thus to develop the proposed age dependent accomplishment probability model, suppose a random sample of size n subjects is drawn from a population of subjects classified into three age groups, namely (1) those aged less than some age a_1; (2) those aged between ages a_1 and a_2; and (3) those aged above some age a_2; where a_1 and a_2 are any non-negative real numbers. Interest is to estimate the probability that a randomly selected subject from the population when in the age interval (1); $0 \le a \le a_1$ `achieves or accomplishes some social and economic status normally expected for this age group; the probability that the

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subject when in the age interval (2); $a_1 \le a < a_2$ achieves or accomplishes some social and economic status normally expected for subjects in this age group; and the probability that the subjects when in the age interval (3); $a \ge a_2$ attains or accomplishes some social and economic status normally expected for subjects in this age group in the population. To estimate these probabilities, let A and $\overline{\text{A}}$ be respectively the events that a randomly selected subject from the population when in the age interval (1), $a \le a_1$, accomplishes and does not accomplish some social and economic status expected for this age group; B and $\overline{\text{B}}$ be respectively the events that the subject when in the age interval (2) $a_1 \le a < a_2$, accomplishes or attains and does not attain some social and economic status expected for subjects of this age group; and C and $\overline{\text{C}}$ be respectively the events that the subjects when in the age interval (3) $a \ge a_2$, accomplishes and does not accomplishes some social and economic status expected for subjects in this age group in the population.

These events would result in the compound events or outcomes in the following sample space among others

$$\text{S} = \left(\text{ABC};\ \text{AB}\overline{\text{C}};\ \text{A}\overline{\text{B}}\text{C};\ \overline{\text{A}}\text{BC};\ \overline{\text{A}}\overline{\text{B}}\text{C};\ \overline{\text{A}}\text{B}\overline{\text{C}};\ \overline{\text{A}}\overline{\text{B}}\text{C};\ \overline{\text{A}}\overline{\text{B}}\overline{\text{C}}\right) \quad (1)$$

To estimate the probabilities of outcomes in the sample space S of Equation (1) and others, we would first need to estimate the probabilities of occurrence of events A, B, and C namely P(A), P(B) and P(C) as well as the probabilities of some conditional events such as P(B|A, P(C|A), PC|B) and PC|AB) etc.

Now to estimate the probabilities (1) P(A), (2) P(B) and (3) P(C) we may let

$$\text{U}_{\text{ij}} = \begin{cases} 1, \text{ if the ith randomly selected subject from the population when in the age interval j or younger accomplish es some social and economic status as expected for subjects in age group j} \\ 0, \text{otherwise} \end{cases} \quad (2)$$

for $i = 1,2,...,n_j$, $\text{j} = 1, 2, 3$ age groups.

Let

$$\Pi_j^+ = \text{P}\left(\text{U}_{\text{ij}} = 1\right)$$

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(3)
And

$$W_j = \sum_{i=1}^{n} U_{ij} \quad (4)$$

Now the expected value and variance of U_{ij} are respectively

$$E(U_{ij}) = \Pi_j^+; \quad Var(U_{ij}) = \Pi_j^+(1-\Pi_j^+) \quad (5)$$

Similarly the expected value and variance of W_j are respectively

$$E(W_j) = \sum_{i=1}^{n}(U_{ij}) = n.\Pi_j^+; \quad Var(W_j) = \sum_{i=1}^{n} Var(U_{ij}) = n.\Pi_j^+(1-\Pi_j^+) \quad (6)$$

Now Π_j^+ is the proportion of or the probability that a randomly selected subject from a sampled population when in the age interval j or younger attains or accomplishes some social and economic status which is normally expected for subjects in the age group j in the population. (Each) sample estimate is

$$\hat{\Pi}_j^+ = p_j = \frac{f_j^+}{n_j} \quad (7)$$

where $f_j^+ = W_j$ is the number of subjects when in one age group j or younger accomplishes or attains some social and economic status normally expected to be attained by individuals in the group j in the population. Note that $f_j^+ = W_j$ is equivalently the total number of 1's in the frequency distribution of the n values of 1s and 0s in U_{ij} for I = 1, 2, …, n; j = 1, 2, 3 age groups.

The sample estimate of the variance of $\hat{\Pi}_j^+$ is from Equation 4

$$Var(\hat{\Pi}_j^+) = \frac{Var(W_j)}{n^2} = \frac{\hat{\Pi}_j^+(1-\hat{\Pi}_j^+)}{n} \quad (8)$$

A null hypothesis that may be of research interest would be that the proportion of subjects or the probability that a randomly selected subject from a population when in age group j or younger attains or accomplishes some social and economic status normally expected for that age group in the population is at least some value Π_{j0} ; or symbolically,

$$Ho: \Pi_j^+ \geq \Pi_{j0}; \text{ versus } H_1; \Pi_j^+ < \Pi_{j0}, \ (0 \leq \Pi_{j0} \leq 1)$$

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(9)

for j = 1, 2, 3 age groups.

The null hypothesis of Equation 9 is tested using the test statistic

$$\chi^2 = \frac{\left(W_j - n.\Pi_{j0}\right)^2}{Var\left(W_j\right)} = \frac{n\left(\hat{\Pi}_j^+ - \Pi_{j0}\right)^2}{\hat{\Pi}_j^+} \quad (10)$$

which under Ho has approximately the chi-square distribution with 1 degree of freedom for sufficiently large n. The null hypothesis of Equation 9 is rejected at α level of significance if

$$\chi^2 \geq \chi^2_{1-\alpha;1} \quad (11)$$

otherwise Ho is accepted.

Note that Equation 7 provides the sample estimate of the required expected accomplishment probability in each of the three age groups in the population. In particularly we have for age groups 1, 2, and 3 respectively that is

$$\hat{\Pi}_1^+ = p_1 = \frac{f_i}{n};\ \hat{\Pi}_2^+ = p_2 = \frac{f_2}{n};\ \ \hat{\Pi}_3^+ = p_2 = \frac{f_3}{n} \quad (12)$$

Now to estimate the probability that a randomly selected subject from the population when in the age interval l or younger, accomplishes some social and economic status normally expected for subjects in this age group l given that this same subject has earlier when in the age interval j $\left(j < l\right)$ accomplished some social and economic status expected for this age group j, we note that the only eligible subjects currently qualified and hence of interest here are only the $n_{l.j} = f_j^+$ subjects who when in the age group j or younger accomplished some social and economic status expected for subjects in this age group j in the population; $j, l = 1, 2, 3;\ j \neq l$. Hence to estimate this probability we may let

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$$U_{il.j} = \begin{cases} 1, & \text{if the ith randomly selected subject in the population when in the age interval or younger } (l) \text{ accomplishes some social and economic status normally expected of subjects in this age group } l \text{ given that the same subject when in the age interval j also accomplished some social and economic status expected of subjects in the age group j in the population.} \\ 0 & \text{otherwise} \end{cases}$$

for $j, l = 1, 2, 3;\ j \neq l$

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Let

$$\Pi^{+}_{l.j} = P\left(U_{l.j} = 1\right)$$

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And

$$W_{l.j} = \sum_{i=1}^{n_{l.j}} U_{l.j}$$

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As usual, the expected value and variance of $U_{il.j}$ are respectively

$$E\left(U_{il.j}\right) = \Pi^{+}_{l.j};\ Var\left(U_{il.j}\right) = \Pi^{+}_{l.j}\left(1 - \Pi^{+}_{l.j}\right)$$

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Similarly the expected value and variance of $W_{l.j}$ are respectively from equation 15

$$E\left(W_{l.j}\right) = \sum_{i=1}^{n_{l.j}} E\left(U_{il.j}\right) = n_{l.j}\Pi^{+}_{l.j};\ Var\left(W_{l.j}\right) = \sum_{i=1}^{n_{l.j}} Var\left(U_{il.j}\right) = n_{l.j}\Pi^{+}_{l.j}\left(1 - \Pi^{+}_{l.j}\right)$$

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Now $\Pi^{+}_{l.j}$ is the proportion of subjects or the probability that a randomly selected subject from a population when in the age interval l or younger accomplishes some social or economic status normally expected of subjects in that age group l given that the same subject when in the age interval j has also accomplished some social and economic status normally expected of subjects in age group in the population, for $l, j = 1,2,3;\ j < l.$ Its sample estimate is

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$$\hat{\Pi}^{+}_{l,j} = \mathrm{p}_{l,j} = \frac{\mathrm{f}^{+}_{l,j}}{\mathrm{n}_{l,j}} \quad (18)$$

Where $\mathrm{f}^{+}_{l,\mathrm{j}} = \mathrm{W}_{l,\mathrm{j}}$ is the number of subjects in the population who when in the age interval l or younger accomplish some social and economic status normally expected of subjects in the age group l given that these same subjects when in the age interval j have also accomplishes some social and economic status normally expected of subjects in age group j in the population. In other words $\mathrm{f}^{+}_{l.j}$ is the total number of 1s in the frequency distribution of the $\mathrm{n}^{+}_{l.j} = \mathrm{f}^{+}_{l.j}$ values of 1s and 0s in $\mathrm{U}_{il.j}$, for $i = 1, 2\text{-}3, ..., \mathrm{n}_{l.j} = \mathrm{f}^{+}_{l.j};\ l.j = 1, 2, 3;\ \ j < l$

The sample variance of $\hat{\Pi}^{+}_{l.j}$ is

$$\mathrm{Var}\left(\hat{\Pi}^{+}_{l.j}\right) = \frac{\mathrm{Var}\left(\mathrm{W}_{l.j}\right)}{\mathrm{n}^{2}_{l.j}} = \frac{\hat{\Pi}^{+}_{l.j}\left(1 - \hat{\Pi}^{+}_{l.j}\right)}{\mathrm{n}_{l.j}} \quad 19$$

A null hypothesis that may be of interest is that the proportion of subjects in a population who are known to have earlier when in age group j have accomplished some social and economic status expected of age group j have also later when in age group l or younger have accomplished some social and economic status expected for that age group l at most some values $\Pi_{l.j0}$; or symbolically

$$\mathrm{Ho}: \Pi^{+}_{l.j} \le \Pi_{l.j0} \ \text{ versus } \ \mathrm{H}_1: \Pi^{+}_{l.j} > \Pi_{l.j0}, \ \ (0 \le \Pi_{l.j0} \le 1) \quad 20$$

which is tested using the test statistics

$$\chi^2 = \frac{\left(\mathrm{W}_{l.j} - \mathrm{n}_{l.j}\Pi_{l.j0}\right)^2}{\mathrm{Var}\left(\mathrm{W}_{l.j}\right)} = \frac{\mathrm{n}_{l.j}\left(\hat{\Pi}^{+}_{l.j} - \Pi_{l.j0}\right)^2}{\hat{\Pi}^{+}_{l.j}\left(1 - \hat{\Pi}^{+}_{l.j}\right)} \quad 21$$

The null hypothesis of Equation 20 is rejected at the α level of significance if Equation 11 is satisfied, otherwise Ho is accepted.

Note that Equation 18 enables the determination of the conditional probabilities that a randomly selected subjects in a population when in the age interval l or younger accomplishes some social and economic status expected for that age group given that the same subject has also when in age interval j accomplished some social and economic status expected of the subject in that age group j in the population, for $l, j = 1,2,3;\ j < l$. Specifically we have that

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$$\hat{P}(B \mid A) = \hat{\Pi}^{+}_{2.1} = p_{2.1} = \frac{f^{+}_{2.1}}{n_{2.1}};\ \ \hat{P}(C \mid A) = \hat{\Pi}^{+}_{3.1} = p_{3.1} = \frac{f^{+}_{3.1}}{n_{3.1}};\ \hat{P}(C \mid B) = \hat{\Pi}^{+}_{3.2} = p_{3.2} = \frac{f^{+}_{3.2}}{n_{3.2}}$$

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where $n_{2.1} = n_{3.1} = f^{+}_{1}$ and $n_{3.2} = f^{+}_{2}$

Finally to estimate the probability that a randomly selected subject from the population when in the age group k or younger accomplishes some social and economic status normally expected for subjects in age group k given that the same subject when in the age interval l has accomplishes some social economic status normally expected for subjects in that age group l and when in the age interval j has also accomplished some social and economic status normally expected for subjects in age group j in the population for $l, j, k = 1, 2, 3;\ j < l < k;$ we note that in this situation , the subjects so qualified and hence of interest here are only those $n_{k.lj} = f^{+}_{l.j}$ subjects who have when in the age interval l and j or younger accomplished some social and economic status normally expected for subjects in both of these two groups l and j in the population, $j < l.$

Hence to estimate this probability we may let

$$U_{ik.lj} = \begin{cases} 1, & \text{if the ith randomly selected subject from the population when in the age interval } k, \text{ or younger accomplish es some social and economic status normally expected of subjects in age group } k, \text{ given that thesame subject has also accomplish ed social and economic status normaly expected of subjects in both of these two age groups, } l \text{ and } j \\ 0, & \text{otherwise} \end{cases}$$

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$$\text{for } i = 1,2,\ldots, n_{k.lj} = f^{+}_{l.j};\ \ l, j, k = 1,2,3;\ j < l < k$$

Let

$$\Pi^{+}_{k.lj} = P(U_{ik.lj} = 1)$$

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and

$$W_{k.lj} = \sum_{i=1}^{n_{k.lj}} U_{ik.lj}$$

25

The expected value and variance of $U_{ik.lj}$ are respectively

$$E(U_{ik.lj}) = \Pi^{+}_{k.lj};\quad Var(U_{ik.lj}) = \Pi^{+}_{k.lj}(1 - \Pi^{+}_{k.lj})$$

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Similarly the expected value and variance of $W_{k.lj}$ are respectively from Equation 25 as

$$E(W_{k.lj}) = n_{k..lj}.\Pi^{+}_{k.lj};\quad Var(W_{k.lj}) = n_{k.lj}\Pi^{+}_{k.lj}(1 - \Pi^{+}_{k.lj})$$

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Now $\Pi^{+}_{k.lj}$ is the proportion of subjects or the probability that a randomly selected subject from the population when in the age interval k, or younger accomplishes some social and economic status normally expected of subjects in age group k given that the same subjects in both age of these two age groups have also accomplished some social and economic status normally expected of subjects in both of these two age groups l and j in the population, for l, j, k $= 1, 2, 3;\quad j < l < k$. Its sample estimate is

$$\hat{\Pi}^{+}_{k.lj} = p_{k.lj} = \frac{f^{+}_{k.lj}}{n_{k.lj}} \qquad 28$$

where $f^{+}_{k.lj} = W_{k.lj}$ is the total number of subjects in the population who when in the age interval k or younger accomplish some social and economic status normally expected of subjects in age group k, given that these same subjects when in the age intervals l and j, have also accomplished some social and economic status normally expected of subjects in both of these two age groups in the population. That is $f^{+}_{k.lj}$ is the total number of 1s in the frequency distribution of the $n^{+}_{k.lj} = f^{+}_{k.lj}$ values of 1s and 0s in $U_{ik.lj}$, for $i = 1, 2, 3, ..., n_{k.lj} = f^{+}_{k.lj};\ l, j, k = 1, 2, 3;\quad j < l < k$

The sample variance of $\hat{\Pi}^{+}_{k.lj}$ is

$$Var\left(\hat{\Pi}^{+}_{k.lj}\right) = \frac{Var\left(W_{k.lj}\right)}{n^{2}_{k.lj}} = \frac{\hat{\Pi}^{+}_{k.lj}\left(1 - \hat{\Pi}^{+}_{k.lj}\right)}{n_{k.lj}} \qquad 29$$

Note that Equation 28 gives the sample estimate of the expected achievement probability in the age interval k, or earlier given accomplishments in the age intervals l and j; $l, j, k = 1,2,3;\ j < l < k.$ in particular we have that

$$\hat{P}\left(C \mid AB\right) = \hat{\Pi}^{+}_{3.21} = P_{3.21} = \frac{f^{+}_{3.21}}{n_{3.21}} \qquad 30$$

where $n_{3.21} = f^{+}_{2.1}$

Again a null hypothesis that may be of interest could be

$$Ho: \Pi^{+}_{k.lj} \geq \theta_0 \quad \text{versus} \quad H_1: \Pi^{+}_{k.lj} < \theta_0 \quad \left(0 \leq \theta_0 \leq 1\right) \qquad 31$$

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which may be tested using the test statistic

$$\chi^2 = \frac{\left(W_{k.lj} - n_{k.lj}.\theta_0\right)^2}{Var\left(W_{k.lj}\right)} = \frac{n_{k.lj}\left(\hat{\Pi}^{+}_{k.lj} - \theta_0\right)^2}{\hat{\Pi}^{+}_{k.lj}\left(1 - \hat{\Pi}^{+}_{k.lj}\right)}$$

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The null hypothesis H_0 of Equation 31 is rejected at the α level of significance if Equation 11 is satisfied, otherwise H_0 is accepted.

With these results we may now proceed using the sample estimated of expected accomplishment probabilities of Equations 7, 18, and 30 to estimate the probabilities of the outcomes or events in Equation 1 and those of other events. For example the probability that a randomly selected subject from the population achieves or accomplishes all expected social and economic status for each of the three age groups in the population that is the probability of events A, B, C is estimated as

$$\hat{P}(ABC) = \hat{P}(C \mid AB).\hat{P}(B \mid A).\hat{P}(A) = P_{3.21}.P_{2.1}P_1$$

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The probability that a subject accomplishes events A and B and not C is estimated as

$$\hat{P}(AB\overline{C}) = \hat{P}(B \mid A).\hat{P}(A) - \hat{P}(ABC) = P_{2.1}.P_1 - P_{3.21}.P_{2.1}.P_1$$

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The probability that the subject accomplishes event C even though events A and B are not accomplished is the probability of the event $C \mid \overline{A}\,\overline{B}$ which is estimated as

$$\hat{P}(C \mid \overline{A}\,\overline{B}) = \frac{\hat{P}(\overline{A}C) - \hat{P}(BC) + \hat{P}(ABC)}{\hat{P}(\overline{A}\,\overline{B})}$$ or when further simplified becomes

$$\hat{P}(C \mid \overline{A}\,\overline{B}) = \frac{P_3.P_{3.1}.P_1 - P_{3.2}.P_2 + P_{3.21}.P_{2.1}.P_1}{1 - P_2 - (1 - P_{2.1}).P_1}$$

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The probability that a randomly selected subject from the population does not accomplish events A, B, C, that is, does not accomplish any of the three social and economic ranks or status in a population is the probability of the event $\overline{A}\,\overline{B}\,\overline{C}$ which is estimated as

$$\hat{P}(\overline{A}\,\overline{B}\,\overline{C}) = 1 - \left(\hat{P}(A) + \hat{P}(B) + \hat{P}(C) - \hat{P}(B \mid A).\hat{P}(A) - \hat{P}(C \mid A).\hat{P}(A) - \hat{P}(C \mid B).\hat{P}(B) + \hat{P}(ABC)\right)$$

Or

$$\hat{P}(\overline{A}\,\overline{B}\,\overline{C}) = 1 - \left(P_1 + P_2 + P_3 - P_{2.1}.P_1 - P_{3.1}.P_1 - P_{3.2}.P_2 + P_{3.21}.P_{2.1}.P_1\right)$$

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The probabilities of other events including those in the sample space S of equation 1 are similarly estimated. The results are presented in Table 1

Table 1 estimates of some age dependent expected accomplishment probabilities in a population

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S/N	Event	Estimates of expected accomplishment probability
1	ABC	$P_{3.21}.P_{2.1}.P_1$
2	$AB\overline{C}$	$P_1.P_{2.1} - P_{3.21}.P_{2.1}.P_1$
3	$A\overline{B}C$	$P_1.P_{3.1} - P_{3.21}.P_{2.1}.P_1$
4	$\overline{A}BC$	$P_2.P_{3.2} - P_{3.21}.P_{2.1}.P_1$
5	$A\overline{B}\,\overline{C}$	$P_1 - P_1.P_{3.1} - P_1.P_{2.1} + P_{3.21}.P_{2.1}.P_1$
6	$\overline{A}B\overline{C}$	$P_2 - P_2.P_{3.2} - P_1.P_{2.1} + P_{3.21}.P_{2.1}.P_1$
7	$\overline{A}\,\overline{B}C$	$P_3 - P_1.P_{3.1} - P_2.P_{3.2} + P_{3.21}.P_{2.1}.P_1$
8	$\overline{A}\,\overline{B}\,\overline{C}$	$1 - \left(P_1 + P_2 + P_3 - P_{2.1}.P_1 - P_{3.1}.P_1 - P_{3.2}.P_2 + P_{3.21}.P_{2.1}.P_1\right)$
9	$BC \mid \overline{A}$	$\dfrac{P_2.P_{3.2} - P_{3.21}.P_{2.1}.P_1}{1 - P_1}$
10	$\overline{B}C \mid A$	$P_{3.1} - P_{3.21}.P_{2.1}$
11	$\overline{C} \mid A\overline{B}$	$\dfrac{P_3 - P_1.P_{3.1} - P_1.P_{2.1} + P_{3.21}.P_{2.1}.P_1}{P_2 - P_{2.1}.P_1}$
12	$\overline{C} \mid \overline{A}B$	$\dfrac{P_2 - P_2.P_{3.2} - P_1.P_{2.1} + P_{3.21}.P_{2.1}.P_1}{P_2 - P_{2.1}.P_1}$
13	$C \mid \overline{A}\,\overline{B}$	$\dfrac{P_3 - P_1.P_{3.1} - P_2.P_{3.2} + P_{3.21}.P_{2.1}.P_1}{1 - P_2 - \left(1 - P_{2.1}\right).P_1}$

ILLUSTRATION

A social worker is interested in determining whether individuals from a certain population who have retired from active work are satisfied with their accomplishments during their more active life. She collected a systematic random sample of retirees from the register of retirees in attendance of a scheduled national conference. The question asked which requires 'yes' or 'no' responses in a questionnaire to be completed by each of the retirees are:
(1)Did you complete your education or some trade apprenticeship before age 25? (a1) (2) Were you able to build or buy and live in your own house before age 50?(a2) (3) if you were to start afresh, would you still be in the same profession or trade? (4) at age 50 (a3) did you look forward to taking a break from active work? These questions are asked because of the, even if contestable, assumption and hence expectation that normally by age 25 one should have been able to complete regular schooling or trade apprenticeship, and by age 50, after working for up to one quarter of a century or 25 years, one should have been able to build and live in own house, so that after age fifty, one should be able to look forward to retirement from relatively active work with no

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regrets about own chosen profession or trade.

In the systematic random sample of possible respondents issued the questionnaire only 60 properly completed and returned these questionnaires.

The responses by these n = 60 retirees who correctly completed the questionnaire are used to illustrate the estimation of expected accomplishment probabilities as proposed. Although one is not obvious of the problems associated with the use of data from a catchment population such as the present one on retirees, which may not be characteristic of the general population with the result that conclusions reached here may not readily apply to the general population, we would nevertheless use the present data, even if for illustrative purposes only.

The responses are presented in Table 2

Table 2: Responses (Yes – Y; No – N) on Self-actualization by a random sample of Retirees

S/No	Complete education or Apprenticeship by age 25 (j)	Lived in own House by age 50 (l)	Happy with own profession or trade and looked forward to retirement at age 50 (k)	S/No	Complete education or Apprenticeship by age 25 (j)	Lived in own House by age 50 (l)	Happy with own profession or trade and looked forward to retirement at age 50 (k)
1	Y	Y	N	31	N	N	N
2	N	Y	Y	32	Y	Y	Y
3	N	N	Y	33	N	N	Y
4	Y	Y	Y	34	N	N	N
5	Y	Y	Y	35	N	N	N
6	N	N	N	36	Y	Y	Y
7	N	N	Y	37	Y	N	Y
8	N	N	N	38	N	N	N
9	Y	N	N	39	Y	N	N
10	N	N	N	40	Y	Y	Y
11	Y	N	N	41	N	N	Y
12	N	Y	Y	42	Y	N	Y
13	Y	N	Y	43	Y	Y	Y
14	Y	Y	Y	44	Y	Y	N
15	Y	N	N	45	N	Y	N
16	N	Y	N	46	Y	N	N
17	N	N	N	47	Y	Y	Y
18	Y	N	Y	48	N	Y	Y
19	Y	N	Y	49	Y	Y	Y
20	N	Y	N	50	N	Y	N
21	N	Y	N	51	N	N	N

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22	Y	N	N	52	N	Y	N
23	Y	Y	N	53	N	N	N
24	Y	Y	Y	54	Y	Y	Y
25	Y	N	Y	55	N	Y	Y
26	Y	N	Y	56	Y	Y	Y
27	Y	N	N	57	N	Y	Y
28	Y	Y	Y	58	N	Y	N
29	Y	N	N	59	Y	N	Y
30	Y	N	Y	60	Y	Y	Y

Using Equation 2 with Table 2 we obtain the frequencies of positive responses (Y) to each of the questions where question number 1 is used as an index of expected accomplishment before age 25; question number 2 is used as an index of expected accomplishment before age 50; and question number 3 and 4 are used as an index of expected accomplishment after taking a break or retiring relatively active work if the responses by the respondent to the two questions are both a "yes" that is a positive response.

The result are shown in Table 3 which also includes other statistics

Table 3 Frequencies of Positive Responses and Other Statistics for the Sample Data of Table 2

Responses

ACCOMPLISHMENT TIME PERIODS			
	Less than 25 years Accomplished Education or trade	25 – 49 years Lived in built own house	50 years or older Happy with own profession or trade and looked forward for retirement
N	60	60	60
f^+	34 $\left(f_1^+\right)$	28 $\left(f_2^+\right)$	33 $\left(f_3^+\right)$
$\hat{\Pi}^+ = p$	0.576 $\left(p_1\right)$	0.467 $\left(p_2\right)$	0.550 $\left(p_3\right)$

It is seen from table 3 that the estimated expected accomplishment probabilities during the time periods < 25 years, (ii) 25 – 49 years, (iii) ≥ 50 years are respectively 0.567, 0.467, and 0.550 respectively.

$$\hat{P}(A) = \hat{\Pi}_1^+ = p_1 = \frac{34}{60} = 0.567;\ \hat{P}(B) = \hat{\Pi}_2^+ = p_2 = \frac{28}{60} = 0.467;\ \hat{P}(C) = \hat{\Pi}_3^+ = p_3 = \frac{33}{60} = 0.550$$

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Hence in the sampled population about 56.7 percent of the population are likely to have completed their education or apprenticeship by age 25 years as expected; only 46.7 percent are able to live in or build own house as expected before age 50 years and over one half (55.0 percent) are still happy with own

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chosen profession or looking forward to retirement as would be expected after age 50 years.

To estimate conditional expected accomplishment probabilities; during a subsequent time period given accomplishment as expected during a previous time period such as P(B|A), P(C|A) and P(C|B) we would use as noted above only those subjects who had earlier accomplished an objective as expected and then determine whether such subjects have also subsequently accomplished some other objective also as expected.

Specifically for the present illustrative example living in or having built own house before age 50 and being happy with own current profession or trade and looking forward to retirement are conditioned on the subject's ability to have completed education or apprenticeship before age 25. Similarly here responses to questions 3 and 4 are only of interest for those subjects who respond 'yes' to question number 2.

The estimation of the resulting conditional expected accomplishment probabilities is shown in table 4

Table 4: Estimation of Conditional accomplishment probability for the sample Data of table 2

S/No. of Subjects responding positive to question No. 1 (j)	Response by subjects to question No 2 (l)	Combined response by subjects to question No. 3 & 4 (k)	S/No of subjects responding positive to question No. 2 (l)	Combined response by subjects to questions 3 & 4 (k)
1	Y	N	1	N
4	Y	Y	2	Y
5	Y	Y	4	Y
9	N	N	5	Y
11	N	N	12	Y
13	N	Y	14	Y
14	Y	Y	16	N
15	N	N	20	N
18	N	Y	21	N
19	N	Y	23	N
22	N	N	24	Y
23	Y	N	28	Y
24	Y	N	32	Y
25	N	Y	36	Y
26	N	Y	40	Y
27	N	N	43	Y
28	Y	Y	44	N

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29	N	N	45	N
30	N	Y	47	Y
32	Y	Y	49	Y
36	Y	Y	50	Y
37	N	Y	52	N
39	N	N	54	Y
40	Y	Y	55	Y
42	N	Y	56	Y
43	Y	Y	57	Y
44	Y	N	58	N
46	N	N	60	Y
47	Y	Y		
49	Y	Y		
54	Y	Y		
56	Y	Y		
59	N	Y		
60	Y	Y		

$n_{i.j} = f_j^+$	$34\,(n_{2.1})$	$34\,(n_{3.1})$	$28\,(n_{3.2})$
$f_{i.j}^+$	$17\,(f_{2.1}^+)$	$22\,(f_{3.1}^+)$	$19\,(f_{3.2}^+)$
$\hat{\Pi}_{i.j}^+ = p_{i.j}$	$0.500\,(p_{2.1})$	$0.647\ (p_{3.1})$	$0.679\,(p_{3.2})$

Using the result of Table 4, we calculate estimated conditional accomplishment probabilities during the three age periods in the population as:

$\hat{P}(B \mid A) = \hat{\Pi}_{2.1}^+ = p_{2.1} \frac{17}{34} = 0.500$; $\hat{P}(C \mid A) = \hat{\Pi}_{3.1}^+ = p_{3.1} \frac{22}{34} = 0.647$;

$\hat{P}(C \mid B) = \hat{\Pi}_{3.2}^+ = p_{3.2} \frac{19}{28} = 0.679$

Finally to calculate estimated conditional accomplishment probability for a given age given accomplishment of objectives by previous ages as expected, we would concentrate, that is be interested on only the responses by subjects on accomplishment by a subsequent age if they have responded positive that is if they have accomplished some objectives by the two previous age-time periods. For example, estimated accomplishment based on question 3 and 4 is here analysed only for subjects who have responded positive that is answered 'yes' to question numbers 1 and 2 as in Table 2

The responses together with the estimated conditional accomplishment probability is shown in table 5

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Table 5: Estimation of Conditional Accomplishment probability by some age given accomplishment as expected in two age periods for the sample data of table 2

S/No of subjects who responds positive to question numbers 1 and 2 (i, j)	Corresponding combined response by subjects to question 3 and 4 (k)
1	N
4	Y
5	Y
14	Y
23	N
24	Y
28	Y
32	Y
36	Y
40	Y
43	Y
44	N
47	Y
49	Y
54	Y
56	Y
60	Y

$n_{k.lj} = f^{+}_{lj}$	$17\,(n_{3.21})$
$f^{+}_{k.lj}$	$14\,(f^{+}_{3.21})$
$\hat{\Pi}^{+}_{k.lj} = p^{+}_{k.lj}$	$0.826\,(p_{3.21})$

It is seen from table 5 that the estimated conditional accomplishment probability for the sampled population after age 50 given accomplishment of set objective before ages 25 and 50 is

$$\hat{P}(C \mid AB) = \hat{\Pi}^{+}_{3.21} = p_{3.21} = \frac{f^{+}_{3.21}}{n_{3.21}} = \frac{14}{17} = 0.824$$

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Thus the probability that a randomly selected subject from the sampled population who had earlier in life before age 25 completed regular

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schooling or trade apprenticeship and was living in or built own house before age 50 years as expected, would be happy with own profession or trade and look forward to retirement from active work after age 50 is estimated to be $\hat{P}(C \mid AB) = 0.824$. That is 82.4 percent of retirees who live in their own house and completed education or apprenticeship in their early twenties would probably be happy with own profession in life and look forward to happy retirement after age 50.

Using Equations 37 – 39 one may now estimate accomplishment probabilities for some possible outcomes or events in table 1 that may result with the proposed accomplishment model.

For example the probability that a randomly selected subject from the sampled population is able to complete education or apprenticeship before age 25 live in or built own house before age 50 and be happy with own profession or trade and look forward to retirement from active work after age 50 as would be expected is estimated using Equations 37 – 39 in equation 33 as

$\hat{P}(ABC) = p_{3.21} \bullet p_{2.1} \bullet p_1 = (0.824)(0.500)(0.567) = 0.234$ =23.4 percent

The probability that a randomly selected subject does not accomplish any of the three expectations during his own life span is estimated Equation 36 as

$\hat{P}(\overline{A}\,\overline{B}\overline{C}) = 1 - (0.567 + 0.467 + 0.550 - (0.567)(0.500) - (0.567)(0.647) - (0.550)(0.679) - 0.234)$

= 1 – (1.818 -1.024) = 1 – 0.794 = 0.206 = 20.6 percent.

In other words, about one fifth of subjects or retirees from the population would be unable to accomplish set objectives as would be expected on retirement from active work.

The probability that a randomly selected object who was unable to active work after age 50 is estimated as:

complete regular education or trade apprenticeship before age 25 and was also unable to build and live in own house before age 50 would if given another opportunity prefer another profession or trade and would not be looking forward to retirement from

$\hat{P}(\overline{C} \mid \overline{A}\,\overline{B}) = \dfrac{\hat{P}(\overline{A}\overline{B}\overline{C})}{1 - \hat{P}(B) - (1 - \hat{P}(B \mid A) \bullet \hat{P}(A))}$ which from Equations 37 – 39 is obtained as

$\hat{P}(\overline{C} \mid \overline{A}\,\overline{B}) = \dfrac{0.206}{1 - 0.467(1 - 0.500)(0.567)} = \dfrac{0.267}{0.249} = 0.827$

In other words about 82.7 percent of subjects or retirees in the sampled population who earlier in life did not complete their education or trade apprenticeship before mid-twenties and could not build and live in their own house before age 50 would likely not be satisfied or happy with their own chosen profession or trade and may not

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likely look forward to retirement from active work after age 50.

The proportion of subjects or retirees who are likely to be happy with own chosen profession and look forward to retirement after age 50 even though they did not complete regular schooling or trade apprenticeship by age 25 but nevertheless were able to build and live in own house before age 50 as expected is estimated as;

$$\hat{P}(C \mid \overline{A}B) = \frac{\hat{P}(\overline{A}BC)}{\hat{P}(\overline{A}B)} = \frac{\hat{P}(C \mid B) \bullet \hat{P}(B) \bullet \hat{P}(ABC)}{\hat{P}(B) - \hat{P}(B \mid A) \bullet \hat{P}(A)}$$

which from Equations 37 – 39 becomes;

$$\hat{P}(C \mid \overline{A}B) = \frac{(0.679)(0.467) - 0.234}{0.467 - (0.500)(0.567)} = \frac{0.083}{0.183} = 0.454$$

Or 45.4 percent, which is less than half of this sub-population of retirees in the sampled population

Other expected accomplishment probabilities are similarly estimated. The results are shown in table 6

Table 6: estimates of expected accomplishment Probabilities for illustrative example of Table 1

S/No	Event Expected accomplishment Probabilities	
1	ABC	0.234
2	$AB\overline{C}$	.050
3	$A\overline{B}C$	0.133
4	$\overline{A}B\overline{C}$	0.083
5	$\overline{A}BC$	0.083
6	$\overline{A}B\overline{C}$	0.100
7	$\overline{A}\overline{B}C$	0.107
8	$\overline{A}\,\overline{B}\overline{C}$	0.206
9	$BC \mid \overline{A}$	0.192
10	$\overline{B}C \mid A$	0.235
11	$\overline{C} \mid AB$	0.530
12	$\overline{C} \mid \overline{A}B$	0.546
13	$\overline{C} \mid \overline{A}\overline{B}$	0.827

It is further seen from Table 6 that the estimated probability that a randomly selected subject or retiree from the population is un able to build and live in own house before age 50 with no regrets for own chosen profession or

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trade given that the same retiree had completed regular education or trade apprenticeship before age 25 is 0.235 or 23.5 percent.

The estimated proportion of retirees from the population who probably would not look forward to retirement or continue with own chosen profession if given another chance after age 50 among those retirees who were able to complete schooling or trade apprenticeship before age 25 but could not build and live in own house before age 50 as would have been expected is 0.530 or about 53 percent, that is over one-half of the sub-population of retirees in the sampled population. This same proportion is however is still higher (54.6 percent) if the retiree did not complete regular education or trade acquisition prior to age 25 even if the retiree had built and lived in own house before age 50 as would normally be expected.

Other expected accomplishment probabilities as desired may be similarly estimated and approximately interpreted.

SUMMARY AND CONCLUSION

This paper assumed that human life span may be partitioned into three epochs by age namely the young in age , relatively dependent age group in which the individual is normally expected to prepare and plan for own role and occupation during the later epochs; the second epoch, the middle working and economically active age group during which the individual is expected to apply the skills acquired during the first epoch in production processes to accomplish some objectives normally expected in this epoch or period of life, and the third and last epoch in later life during which the individual is relatively less economically active and is expected to normally be able to relax probably relishing own accomplishments during the two previous epochs as would normally be expected but also sometimes having some regrets with the individual wishing that he or she had done certain things differently during the previous epochs of own life.

Under these assumptions, a statistical model of expected accomplishment probabilities in a population was developed including methods for the estimation of these probabilities during each of the three age epochs. Test statistics were also developed for testing desired null hypotheses about expected accomplishment probabilities.

The proposed method was illustrated with some sample data on retirees from a certain population in which human age span was partitioned into three, namely, less than 25 years; 25 to 50 years; and over 50 years.

Using these sample data, it was found that less than one fourth of retirees from the sampled population were able to accomplish all that was normally expected of subjects from the population after retirement from relatively active work. However only about one-fifth of retirees did not seem to have accomplished all that was

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normally expected of. Among the retirees who did not accomplish what was normally expected of them during the first epoch of own life and also failed in their middle ages or second epoch to accomplish as expected, over four-fifths were found not to be satisfied with own profession or trade and were not looking forward to retirement from active work as would normally be expected during the third and last epoch of life.

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